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This gives the equation of condition

$$x'^2 + y'^2 - ax' + \frac{ab - b^2 - c^2}{c} y' = 0.$$

The coördinates of O, A and B satisfy this equation. Therefore this is the circumscribed circle, and the circumscribed circle is the required locus.

II. Solution by William Hoover, Columbus, Ohio, and Otto Dunkel, Washington University.

Take the given triangle, ABC, as the triangle of reference in trilinear coördinates, the side opposite A being $\alpha = 0$; also $(\alpha', \beta', \gamma')$ the point from which the perpendiculars are drawn. Take any point in $\alpha = 0$, as $(0, \beta_1, \gamma_1)$; then the straight line through $(\alpha', \beta', \gamma')$ and $(0, \beta_1, \gamma_1)$ is given by

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha' & \beta' & \gamma' \\ 0 & \beta_1 & \gamma_1 \end{vmatrix} = 0$$

or,

$$\alpha(\beta'\gamma_1 - \gamma'\beta_1) - \beta\alpha'\gamma_1 + \gamma\alpha'\beta_1 = 0.$$

This is perpendicular to $\alpha = 0$ if

$$\beta'\gamma_1 - \gamma'\beta_1 - \alpha'\beta_1 \cos B + \alpha'\gamma_1 \cos C = 0$$

or

$$-(\gamma' + \alpha' \cos B)\beta_1 + (\beta' + \alpha' \cos C)\gamma_1 = 0;$$

so that we can write for the coördinates of the foot of the perpendicular from $(\alpha', \beta', \gamma')$ upon $\alpha = 0$,

$$0, \quad \beta' + \alpha' \cos C, \quad \gamma' + \alpha' \cos B.$$

In the same way we get the feet of the other two perpendiculars.

These three points will be on a straight line if

$$\begin{vmatrix} 0 & \beta' + \alpha' \cos C & \gamma' + \alpha' \cos B \\ \alpha' + \beta' \cos C & 0 & \gamma' + \beta' \cos A \\ \alpha' + \gamma' \cos B & \beta' + \gamma' \cos A & 0 \end{vmatrix} = 0.$$

This reduces to [1]

$$(\alpha' \sin A + \beta' \sin B + \gamma' \sin C)(\beta'\gamma' \sin A + \gamma'\alpha' \sin B + \alpha'\beta' \sin C) = 0.$$

The first factor is equal to the area of the triangle divided by the radius of the circumscribed circle and hence cannot be zero. The second factor put equal to zero gives the equation of the circumscribed circle. Hence the circumscribed circle comprises the entire locus.

A simple proof may be obtained by reversing the reasoning given on page 118 of Salmon's *Conic Sections*, edition 6 (London, 1879).

If we assume Wallace's theorem the above may be regarded as a derivation of the equation of the circumscribed circle; for it is the equation of a conic which includes all the points of this circle.

Also solved by T. L. Bennett, A. M. Harding, Arthur Pelletier, and H. L. Olson.

2906 [1921, 277]. Proposed by ELIJAH SWIFT, University of Vermont.

Given any number of five digits, reverse the order and subtract the smaller of the two numbers thus formed from the larger. Show that if told the last three digits of this difference, we can find the first two, and give a simple rule for determining them.

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[1] If we multiply the three rows by $\beta'\gamma' \sin A$, $\gamma'\alpha' \sin B$ and $\alpha'\beta' \sin C$, respectively, and add, we shall get a new row whose elements are α', β', and γ' times the expression

$$\beta'\gamma' \sin A + \gamma'\alpha' \sin B + \alpha'\beta' \sin C,$$

which is, therefore, a factor.

SOLUTION BY A. V. RICHARDSON, Bishop's College, Lennoxville, Quebec, Canada.

We may indicate the subtraction as follows:

$$\begin{array}{ccccc} a & b & c & d & e \\ e & d & c & b & a \\ \hline x & y & m & n & p \end{array}$$

where the letters denote the digits of the numbers. Since the first number is the larger, either $a > e$ or $a = e$ and $b > d$. The digits of the difference will be as in the following table.

	x	y	m	n	p
$a > e,\ b > d$	$a - e$	$b - d - 1$	9	$9 + d - b$	$10 + e - a$
$b = d$	$a - e - 1$	9	9	9	$10 + e - a$
$b < d$	$a - e - 1$	$10 + b - d$	0	$d - b - 1$	$10 + e - a$
$a = e,\ b > d$	0	$b - d - 1$	9	$10 + d - b$	0

From the table we may formulate the following rule:

$$x = 10 - p \quad \text{and} \quad y = 8 - n$$

except

(1) when $p = 0$, and then $x = 0$ and $y = 9 - n$;
(2) when $m = 0$, and then $x = 9 - p$, and $y = 9 - n$;
(3) when $m = n = 9$, $p \neq 0$, and then $x = 9 - p$ and $y = 9$.

Also solved by T. M. BLAKSLEE, H. N. CARLTON, A. M. HARDING, L. C. MATHEWSON, C. J. STOWELL, F. L. WILMER.

NOTES AND NEWS.

It is hoped that readers of the MONTHLY will coöperate in contributing to the general interest of this department by sending items to R. W. BURGESS, Brown University, Providence, R. I.

In the temporary assignment of Mr. E. B. MODE to duties connected with University administration, the following have been appointed temporary instructors in the College of Liberal Arts, Boston University: C. E. HAIGLER of the Rehabilitation Division, Franklin Union; J. A. MARSH of the English High School, Boston; and R. W. GARDNER of the Eastern Nazarene College, Wollaston.

At the University of Michigan, Professor W. H. BUTTS has retired after serving twenty-four years as instructor and professor of mathematics and fourteen years as assistant dean of the Colleges of Engineering and Architecture. He has been appointed professor emeritus.

Mr. L. J. COMRIE, M.A., F.R.A.S., of New Zealand and St. John's College, University of Cambridge, Cambridge, England, is in residence at the Sproul Observatory of Swarthmore College as Research Assistant for the coming year. Mr. Comrie is the holder of the Isaac Newton Studentship awarded in April 1921 at Cambridge University for research in Astronomy. He has been working under Professor A. S. Eddington for three years and has made a special study of occultations of stars by planets. He will continue this study and in particular the publications of predictions of these occultations. He was successful in organizing a Computing Section of the British Astronomical Association, and also edited the first edition of that Association's *Observer's Handbook*, published in 1922. He will participate in the researches upon the parallaxes of stars which is the prime work of the Sproul Observatory.